Problem Set 2: Matrix and Linear Algebra Operations in Python and Numpy
Computational Perception and Artificial Intelligence

Description:

Solve each problem using Python and Numpy. You will need to do all the work in a single python script called `ps02.py`. For this example, the outline of code has been provided showing the required style for comments and outputs. You will submit your work in a zipped file called `ps02.zip` emailed to Mr. Michaud by the due date.

The purpose of this assignment is to practice creating and manipulating 2D matrices using Python and Numpy. Consult the class notes and online sources. However, make sure you do your own work with coding. Do not merely copy answers. Work to understand each matrix operation as these will provide the foundation for vision, perception, and intelligence algorithms.

Setup:

A. Download and unzip the following folder into your Lastname Perception Directory

http://nebomusic.net/perception/ps02.zip

B. Open the `ps02.py` file with IDLE and place your name in the comments as indicated in code.

C. Take note of the format for the `ps02.py` file. Future problem sets will need to be coded in this style.

Questions: (Question 1 is done in the code as an example)

1. Create the following matrices A and B as shown below with uint32 datatypes. Calculate the matrix C as an element wise addition of A and B:

A + B = C

$$A = \begin{bmatrix} 3 & 6 & 8 \\ 7 & 4 & 1 \\ 2 & 6 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 9 & 7 & 4 \\ 3 & 5 & 6 \\ 6 & 3 & 1 \end{bmatrix}$$

Print matrix C to the console (sample for 1 has been done to show the style)

2. Create the following matrices A and B as shown below as float32 datatype. Calculate the matrix C as an element wise subtraction of A minus B:

A-B = C

$$A = \begin{bmatrix} 4.5 & 7.8 & 3.1 \\ 2.7 & 1.4 & 5.5 \\ 3.2 & 5.6 & 8.7 \end{bmatrix} \quad B = \begin{bmatrix} 9.8 & 7.5 & 3.9 \\ 2.9 & 9.2 & 0.5 \\ 6.5 & 8.4 & 8.3 \end{bmatrix}$$

Print matrix C to the console

3. Create the following matrices A and B as shown below as float 32 datatype. Calculate the dot product C:

AB = C

$$A = \begin{bmatrix} 4 & 7 & 9 \\ 6 & 3 & 2 \\ 1 & 5 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 6 & 2 \\ 4 & 6 & 7 \\ 5 & 4 & 2 \end{bmatrix}$$

Print matrix c to the console

4. Create the following matrix A as shown below with float 32 datatype. Print the sum of all elements of matrix A to the console.

$$A = \begin{bmatrix} 10 & 13 & 15 \\ 31 & 37 & 29 \\ 11 & 53 & 83 \end{bmatrix}$$

5. Create the following matrix A as shown below with float 32 datatype. Make B equal to the A^T. Make C equal to A^{-1}. Print B and C to the console.

$$A = \begin{bmatrix} 4 & 8 & 8 \\ 6 & 3 & 1 \\ 9 & 5 & 8 \end{bmatrix}$$

6. Matrix M is already declared in the code. Create matrix N as a partial of Matrix M with rows 3 to 7 and columns 4 to 9. Use submatrix notation. (No credit if you do this 'by hand') Print matrix N to the console.

7. Solve the following system of equations for x, y, and z using matrix operations: Print the values of x, y, and z to the console.

$$\begin{aligned} 7x + 5y - 3z &= 16 \\ 3x - 5y + 2z &= -8 \\ 5x + 3y - 7z &= 0 \end{aligned}$$

8. Solve for x1, x2, x3, x4 in this system of equations. Print values of x1, x2, x3, x4 to the console:

$$\begin{aligned} x_1 + x_2 \qquad + x_4 &= 2, \\ 2x_1 + x_2 - x_3 + x_4 &= 1, \\ 4x_1 - x_2 - 2x_3 + 2x_4 &= 0, \\ 3x_1 - x_2 - x_3 + 2x_4 &= -3. \end{aligned}$$

9. Solve the following problem. Print the Values of A, B, and C to the console

The standard equation of a circle is $x^2+y^2+Ax+By+C=0$. Find the equation of the circle that passes through the points
$(1,1)$

,
$(2,-4)$

, and
$(5,5)$.

10. Solve the problem below. Write your answers as comments in the code.

Suppose that in a biological system there are n species of animals and m sources of food. Let x_j represent the population of the jth species for each $j = 1, \ldots, n$; b_i represent the available daily supply of the ith food; and a_{ij} represent the amount of the ith food consumed on the average by a member of the jth species. The linear system

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1, \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2, \\ \vdots \quad\quad & \quad\quad \vdots \quad\quad \vdots \quad\quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m, \end{aligned}$$

represents an equilibrium where there is a daily supply of food to precisely meet the average daily consumption of each species.

a. Let $A = (a_{ij}) = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 1 & 0 & 2 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$, $\mathbf{x} = (x_j) = [1000, 500, 350, 400]$, and $\mathbf{b} = (b_i) =$ $[3500, 2700, 900]$. Is there sufficient food to satisfy the average daily consumption?

b. What is the maximum number of animals of each species that could be individually added to the system with the supply of food still meeting the consumption?

c. If species 1 became extinct, how much of an individual increase of each of the remaining species could be supported?

d. If species 2 also became extinct, how much of an individual increase of each of the remaining species could be supported?